Generation + Propagation Rules

$$\frac{x :_\lambda \tau \in \Gamma}{\Gamma \vdash_\Downarrow x : \sigma \rightsquigarrow \tau \sim \sigma} \textsc{VarCon}_\lambda$$

$$\frac{x : \sigma_1 \in \Gamma}{\Gamma \vdash_\Downarrow x : \sigma_2 \rightsquigarrow \mathsf{InstanceOf}(\sigma_1, \sigma_2)} \textsc{VarCon}$$

$$\frac{\Gamma \vdash_\Downarrow e : \tau \rightsquigarrow C}{\Gamma \vdash_\Downarrow e : \forall \overline{a}.\, Q \Rightarrow \tau \rightsquigarrow \forall \overline{a}.(Q \supset C)} \textsc{Prop}\forall$$

$$\frac{\alpha,\, \beta \text{ fresh} \qquad \Gamma, x :_\lambda \alpha \vdash_\Downarrow e : \beta \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda x.e : \gamma \rightsquigarrow C \wedge \gamma \sim \alpha \to \beta} \textsc{AbsVar}$$

$$\frac{\Gamma, x :_\lambda \alpha \vdash_\Downarrow e : \sigma_2 \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda x.e : \alpha \to \sigma_2 \rightsquigarrow C} \textsc{AbsArrowVar}$$

$$\frac{\Gamma, x : \forall a.\sigma_1 \vdash_\Downarrow e : \sigma_2 \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda x.e : (\forall a.\sigma_1) \to \sigma_2 \rightsquigarrow C} \textsc{AbsArrow}\forall$$

$$\frac{\sigma_1 \not\equiv \alpha \qquad \sigma_1 \not\equiv \forall a.\sigma \qquad \Gamma, x :_{\{\lambda,\text{-}\}} \sigma_1 \vdash_\Downarrow e : \sigma_2 \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda x.e : \sigma_1 \to \sigma_2 \rightsquigarrow C} \textsc{AbsArrowRest}$$

$$\frac{\alpha \text{ fresh} \qquad \Gamma, x : \sigma_1 \vdash_\Downarrow e : \alpha \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda(x :: \sigma_1).e : \gamma \rightsquigarrow C \wedge \gamma \sim \sigma_1 \to \alpha} \textsc{AbsAVar}$$

$$\frac{\Gamma, x : \sigma_1 \vdash_\Downarrow e : \sigma_3 \rightsquigarrow C}{\Gamma \vdash_\Downarrow \lambda(x :: \sigma_1).e : \sigma_2 \to \sigma_3 \rightsquigarrow C \wedge \sigma_2 \sim \sigma_1} \textsc{AbsAArrow}$$

$$\frac{f_1 \ldots f_m : \forall \overline{a}.\, \sigma_1 \to \cdots \to \sigma_n \to \sigma_r \in \Gamma \qquad \theta = [\overline{a \to \alpha}] \qquad \Gamma \vdash_\Downarrow e_i : \theta(\sigma_i) \rightsquigarrow C_i}{\Gamma \vdash_\Downarrow f_1 \ldots f_m\, e_1 \ldots e_n : \sigma \rightsquigarrow \bigwedge_i C_i \wedge \theta(Q) \wedge \mathsf{InstanceOf}(\theta(\sigma_r), \sigma)} \textsc{AppFun}$$

$$\frac{\Gamma \vdash_\Downarrow e_1 : \alpha \to \sigma \rightsquigarrow C_1 \qquad \Gamma \vdash_\Downarrow e_2 : \alpha \rightsquigarrow C_2 \qquad \alpha \text{ fresh}}{\Gamma \vdash_\Downarrow e_1\, e_2 : \sigma \rightsquigarrow C_1 \wedge C_2} \textsc{App}$$

$$\frac{\Gamma \vdash_\Downarrow e : \sigma_1 \rightsquigarrow C}{\Gamma \vdash_\Downarrow (e :: \sigma_1) : \sigma_2 \rightsquigarrow C \wedge \mathsf{InstanceOf}(\sigma_1, \sigma_2)} \textsc{Annot}$$

$$\frac{\alpha \text{ fresh} \qquad \Gamma, x :_\lambda \alpha \vdash_\Downarrow e_1 : \alpha \rightsquigarrow C_1 \qquad \Gamma, x :_\lambda \alpha \vdash_\Downarrow e_2 : \sigma \rightsquigarrow C_2}{\Gamma \vdash_\Downarrow \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 : \sigma \rightsquigarrow C_1 \wedge C_2}\ \textsc{Let}$$

$$\frac{\Gamma, x : \sigma_1 \vdash_\Downarrow e_1 : \sigma_1 \rightsquigarrow C_1 \qquad \Gamma, x : \sigma_1 \vdash_\Downarrow e_2 : \sigma_2 \rightsquigarrow C_2}{\Gamma \vdash_\Downarrow \mathsf{let}\ x :: \sigma_1 = e_1\ \mathsf{in}\ e_2 : \sigma_2 \rightsquigarrow C_1 \wedge C_2}\ \textsc{LetA}$$

$$\frac{\begin{array}{l}\overline{\gamma} \text{ fresh} \qquad \Gamma \vdash_\Downarrow e : \mathsf{T}\,\overline{\gamma} \rightsquigarrow C \\ \text{for each branch } K_i\,\overline{x_i} \to u_i \text{ do} \\ \quad K_i : \forall \overline{a}\overline{b_i}.Q_i \Rightarrow \overline{v_i} \to \mathsf{T}\,\overline{a} \in \Gamma \qquad \overline{b_i} \text{ fresh} \\ \quad \Gamma, \overline{x_i : [\overline{a \mapsto \gamma}]v_i} \vdash_\Downarrow u_i : \sigma \rightsquigarrow C_i \\ \quad \overline{\delta_i} = fuv(\sigma, C_i) - fuv(\Gamma, \overline{\gamma}) \\ \quad C_i' = \begin{cases} C_i & \text{if } \overline{b_i} \text{ and } Q_i \text{ empty} \\ \forall \overline{\delta_i}.\,([\overline{a \mapsto \gamma}])Q_i \supset C_i) \end{cases}\end{array}}{\Gamma \vdash_\Downarrow \mathsf{case}\ e\ \mathsf{of}\ \{\overline{K_i\,\overline{x_i} \to u_i}\} : \sigma \rightsquigarrow C \wedge \bigwedge C_i'}\ \textsc{Case}$$